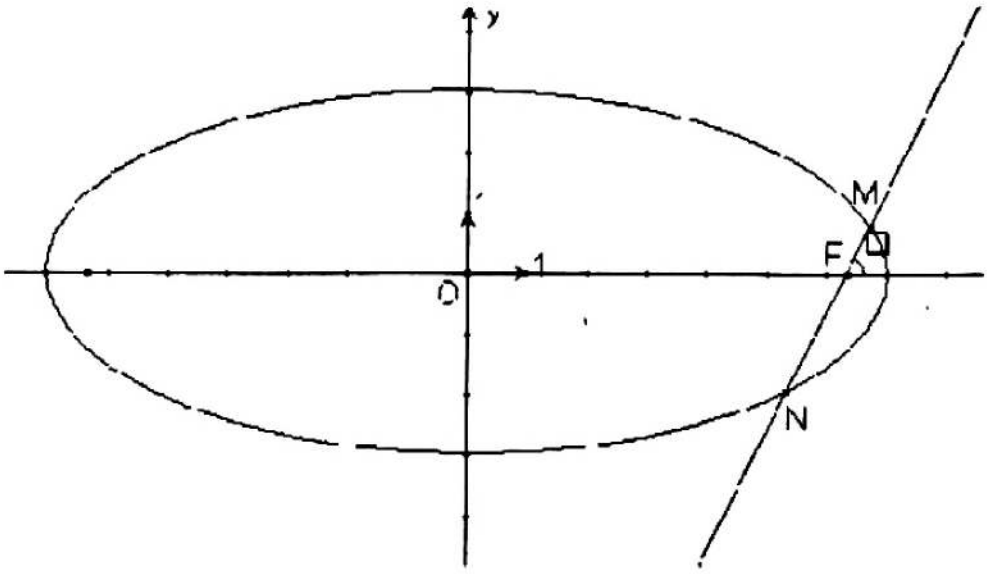
Exercice 17



F(c , 0) $a^2 = b^2 + c^2$

dans le repère $(F, \vec{i}, \vec{j})$

$$(E): \frac{(x+c)^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$(E): b^2(x+c)^2 + a^2.y^2 = a^2.b^2$$

$$(E): a^2(x^2+y^2) = (b^2 - cx)^2$$

soit $r = FM$; $\theta = \left(\vec{i}; \overrightarrow{FM}\right)[2\pi]$ et $r' = FN$; $\left(\vec{i}; \overrightarrow{FN}\right) \equiv \theta + \pi[2\pi]$

Dans le repère $(F, \vec{i}, \vec{j})$:

$$M(r.\cos\theta, r.\sin\theta)$$

$$N(r'.\cos(\theta+\pi), r'.\sin(\theta+\pi))$$

$$M \in (E) \Rightarrow a^2(r^2.\cos^2\theta + r^2.\sin^2\theta) = (b^2 - cr.\cos\theta)^2$$

$$\Rightarrow a^2r^2 = (b^2 - cr.\cos\theta)^2$$

$$\Rightarrow ar = b^2 - cr.\cos\theta \quad \text{ou} \quad ar = -b^2 + cr.\cos\theta$$

$$\Rightarrow r = \frac{b^2}{a + c.\cos\theta} \quad \text{ou} \quad r = \frac{-b^2}{a - c.\cos\theta} < 0$$

d'où

$$\boxed{r = FM = \frac{b^2}{a + c.\cos\theta}}$$

$$r' = FN = \frac{b^2}{a + c.\cos(\theta+\pi)} = \frac{b^2}{a - c.\cos\theta}$$

$$\frac{1}{r} + \frac{1}{r'} = \frac{2a}{b^2}$$

Qarray.tn -Aqra Blech- Qarray.tn